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EVEN VERTEX EQUITABLE EVEN LABELING

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Abstract: Let G be a graph with p vertices and q edges and $A = \{0,2,4,\ldots,q+1\}$ if q is odd or $A = \{0,2,4,\ldots,q\}$ if q is even. A graph G is said to be an even vertex equitable even labeling if there exists a vertex labeling $f: V(G) \rightarrow A$ that induces an edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all edges uv such that for all a and b in A, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $2,4,\ldots,2q$, where $v_f(a)$ be the number of vertices v with $f(v) = a$ for $a \in A$. A graph that admits even vertex equitable even labeling is called an even vertex equitable even graph. In this paper, we prove that path, comb, complete bipartite, cycle, $K_2 + mK_1$, bistar, ladder, $S(L_n)$,$S(B_{n,n})$ and $L_n \odot K_1$ are even vertex equitable even graphs.

Keywords: vertex equitable labeling, even vertex equitable even labeling,

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1. INTRODUCTION

All graphs considered here are simple, finite, connected and undirected. Let $G(V,E)$ be a graph with p vertices and q edges. We follow the basic notations and terminologies of graph theory as in [2]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions and a detailed survey of graph labeling can be found in[1]. The concept of vertex equitable labeling was introduced by Lourdusamy and Seenivasan [3].

Definition 1.1[3]Let$G$ be a graph with p vertices and q edges and $A = \left\{0,1,2,\dots,\left\lceil\frac{q}{2}\right\rceil\right\}$. A graph G is said to be vertex equitable if there exists a vertex labeling $f: V(G) \to A$ that induces an edge labeling f^*defined by $f^*(uv) = f(u) + f(v)$ for all edges uv such that for all a and b in A, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $1,2,3 \dots, q$,where $v_f(a)$ be the number of vertices v with $f(v) = a$ for $a \in A$.A graph that admits vertex equitable labeling is called vertex equitable graph.

Motivated by the concept of vertex equitable labeling, we introduce a new concept of vertex equitable labeling called even vertex equitable even labeling.

Definition 1.2: Let G be a graph with p vertices and q edges and $A = \{0,2,4,\dots,q+1\}$ if q is odd or $A = \{0,2,4,\dots,q\}$ if q is even. A graph G is said to be an even vertex equitable even labeling if there exists a vertex labeling $f: V(G) \to A$ that induces an edge labeling f^*defined by $f^*(uv) = f(u) + f(v)$ for all edges uv such that for all a and b in A, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $2,4,\dots,2q$, where $v_f(a)$ be the number of vertices v with $f(v) = a$ for $a \in A$. A graph that admits even vertex equitable even labeling is called an even vertex equitable even graph.

Definition 1.3: The *corona* $G_1 \odot G_2$ of two graphs $G_1(p_1,q_1)$ and $G_2(p_2,q_2)$ is defined as the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of $G_1$with an edge to every vertex in the i^{th} copy of G_2.

Definition 1.4: The *bistar* $B_{n,n}$ is the graph obtained by attaching the apex vertices of two copies of $K_{1,n}$ by an edge.

Definition 1.5: The *complete bipartite* graph is a simple bipartite graph such that every vertex in one of the bipartition subsets is joined to every vertex in the other bipartition subset. Any complete bipartite graph that has m vertices in one of its subsets and n vertices in other is denoted by $K_{n,m}$.

Definition 1.6: The join of two graphs G_1 and G_2 is denoted by $G_1 + G_2$ and whose vertex set is $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and edge set is
$E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv: u \in V(G_1), v \in V(G_2)\}$.

Definition 1.7: The *subdivision of graph* $S(G)$ is obtained from G by subdividing each edge of G with a vertex.

Definition 1.8: A Cartesian product of two graphs G_1 and G_2 is the graph $G_1 \times G_2$ such that its vertex set is a cartesian product of $V(G_1)$ and $V(G_2)$ i.e.,
$V(G_1 \times G_2) = V(G_1) \times V(G_2) = \{(x, y): x \in V(G_1), y \in V(G_2)\}$ and its edge set is defined as
$(G_1 \times G_2) = \{((x_1, x_2), (y_1, y_2)) : x_1 = y_1 \; and \; (x_2, y_2) \in E(G_2) \; or \; x_2 = y_2 \; and \; (x_1, y_1) \in E(G_1)\}$
.

Definition 1.9: Thegraph $P_n \times P_2$ is called a *ladder graph*.

2. MAIN RESULTS

Theorem 2.1 The path P_n is an even vertex equitable even graph.

Proof: Let P_n be a path with consecutive vertices $u_1, u_2, \dots, u_n$. Then P_n is of order n and size $n-1$. Define $f: V(P_n) \to A = \begin{cases} 0,2,4, \dots, n & if \;\; n-1 \;\; is \; odd \\ 0,2,4, \dots, n-1 & if \;\; n-1 \;\; is \;\; even \end{cases}$ as follows:

$$f(u_i) = \begin{cases} i-1 & if \;\; i \;\; is \; odd \\ i & if \;\; i \;\; is \; even \end{cases} \text{ for } 1 \le i \le n.$$

It is easily verified that the induced edge labels of P_n are $2,4,6 \dots, 2n-2$.

Thus, $|v_f(a) - v_f(b)| \le 1$ for all $a, b \in A$.

Hence the path P_n is an even vertex equitable even graph.

Example 2.2 An even vertex equitable even labeling of P_6 is shown in Figure 2.1

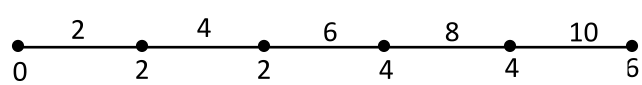

Figure 2.1

Theorem 2.3 The comb is an even vertex equitable even graph.

Proof: Let G be a comb obtained from the path $u_1, u_2, \ldots, u_n$ by joining a vertex v_i to u_i for each $i = 1,2,\ldots,n$. Then G is of order $2n$ and size $2n-1$.

Define $f: V(G) \to A = \{0,2,4,6,\ldots,2n\}$ as follows:

$$f(u_i) = \begin{cases} 2i & \text{if } i \text{ is odd} \\ 2i-2 & \text{if } i \text{ is even} \end{cases} ; 1 \le i \le n$$

$$f(v_i) = \begin{cases} 2i-2 & \text{if } i \text{ is odd} \\ 2i & \text{if } i \text{ is even} \end{cases} ; 1 \le i \le n$$

It can be easily verified that the induced edge labels of $P_n \odot K_1$ are $2,4,6,\ldots,4n-2$.

Thus, $|v_f(a) - v_f(b)| \le 1$ for all $a, b \in A$.

Hence the comb is an even vertex equitable even graph.

Example 2.4 An even vertex equitable of comb is shown in Figure 2.2

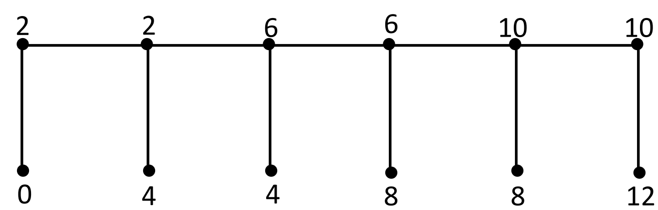

Figure 2.2

Theorem 2.5 The cycle C_n is an even vertex equitable even graph if and only if $n \equiv 0,3(mod\ 4)$

Proof: Let C_n be a cycle with consecutive vertices $u_1, u_2, u_3, \ldots, u_n$, where $n \equiv 0,3(mod\ 4)$. Then C_n is of order n and size n.

Define $f: V(C_n) \to A = \begin{cases} 0,2,4,\ldots,n+1 & \text{if } n \text{ is odd} \\ 0,2,4,\ldots,n & \text{if } n \text{ is even} \end{cases}$ as follows:

$$f(u_i) = \begin{cases} i-1 & \text{if } i \text{ is odd and } 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor \\ i+1 & \text{if } i \text{ is odd and } \left\lceil \frac{n}{2} \right\rceil \le i \le n \\ i & \text{if } i \text{ is even and } 1 \le i \le n \end{cases}$$

It can be easily verified that the induced edge labels of cycle are $2,4,6,\ldots,2n$.

Thus, $|v_f(a) - v_f(b)| \leq 1$ for all $a, b \in A$.

Hence the cycle C_n is an even vertex equitable even graph iff $n \equiv 0,3 (mod\ 4)$.

Theorem 2.6 The graph $K_{2,n}$ is an even vertex equitable even graph.

Proof: Let (V_1, V_2) be the bipartition of $K_{2,n}$ with $V_1 = \{u, v\}$ and $V_2 = \{v_1, v_2, \ldots, v_n\}$. Then $K_{2,n}$ is of order $n + 2$ and size $2n$.

Define $f: V(K_{2,n}) \rightarrow A = \{0,2,4 \ldots, 2n\}$ as follows:

$$f(v) = 0; \ \ f(u) = 2n;$$

$$f(v_i) = 2i \, ; 1 \leq i \leq n.$$

It can be easily verified that the induced edge labels of $K_{2,n}$ are $2,4, \ldots, 4n$.

Thus, $|v_f(a) - v_f(b)| \leq 1$ for all $a, b \in A$.

Hence the graph $K_{2,n}$ is an even vertex equitable even graph.

Theorem 2.7 The bistar$B_{n,n}$ is an even vertex equitable even graph.

Proof: Let u and v be the end vertices of K_2 and for each $i = 1,2, \ldots, n$. Let u_i, v_i be the vertices adjacent to u and v respectively. Then $B_{n,n}$ is of order $2n + 2$ and size $2n + 1$. Define $f: V(B_{n,n}) \rightarrow A = \{0,2,4, \ldots, 2n + 2\}$ as follows:

$$f(u) = 0 \, ; f(v) = 2n + 2 \, ;$$

$$f(v_i) = f(u_i) = 2i \, ; 1 \leq i \leq n$$

It can be easily verified that the induced edge labels of $B_{n,n}$ are $2,4,6, \ldots, 4n + 2$.

Thus, $|v_f(a) - v_f(b)| \leq 1$ for all $a, b \in A$.

Hence the bistar $B_{n,n}$ is an even vertex equitable even graph.

Example 2.8 An even vertex equitable even labeling of $B_{5,5}$ is shown in Figure 2.3

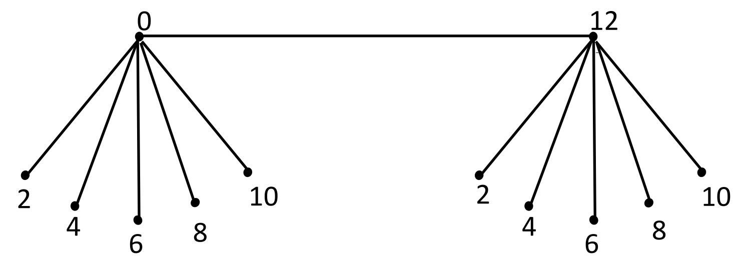

Figure 2.3

Theorem 2.9 The graph $K_2 + mK_1$ is an even vertex equitable even graph.

Proof: Let$G = K_2 + mK_1$. Let $V(G) = \{u, v, w_1, w_2, \dots, w_m\}$ and $E(G) = \{uv\} \cup \{uw_i, vw_i : 1 \le i \le m\}$. Then G is of order $m + 2$ and size $2m + 1$.

Define $f: V(G) \to A = \{0,2,4, \dots ,2m + 2\}$ as follows:

$$f(u) = 0;$$

$$f(v) = 2m + 2;$$

$$f(w_i) = 2i \ ; 1 \le i \le m$$

It can be easily verified that the induced edge labels of $K_2 + mK_1$ are $2,4, \dots ,4m + 2$.

Thus,$|v_f(a) - v_f(b)| \le 1$ for all $a, b \in A$.

Hence the graph $K_2 + mK_1$ is an even vertex equitable even graph.

Example 2.10 An even vertex equitable even labeling of $K_2 + 5K_1$ is shown in Figure 2.4

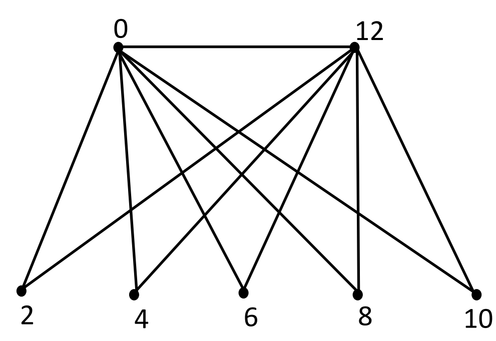

Figure 2.4

Theorem 2.11 The ladder graph L_n is an even vertex equitable even graph.

Proof: Let $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ be the vertices of L_n.

Let $E(L_n) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \le i \le n-1\} \cup \{u_i v_i : 1 \le i \le n\}$. Then L_n is of order $2n$ and size $3n-2$.

Define $f: V(L_n) \to A = \begin{cases} 0,2,4,\dots,3n-1 & \text{if } 3n-2 \text{ is odd} \\ 0,2,4 \dots,3n-2 & \text{if } 3n-2 \text{ is even} \end{cases}$ as follows:

$$f(u_i) = \begin{cases} 3i-1 & \text{if } i \text{ is odd} \\ 3i-2 & \text{if } i \text{ is even} \end{cases} ; 1 \le i \le n.$$

$$f(v_i) = \begin{cases} 3i-3 & \text{if } i \text{ is odd} \\ 3i-2 & \text{if } i \text{ is even} \end{cases} ; 1 \le i \le n.$$

It can be easily verified that the induced labels of L_n are $2,4,\dots,6n-4$.

Thus, $|v_f(a) - v_f(b)| \le 1$ for all $a, b \in A$.

Hence the graph L_n is an even vertex equitable even graph.

Example 2.12 An even vertex equitable even labeling of L_5 is shown in Figure 2.5

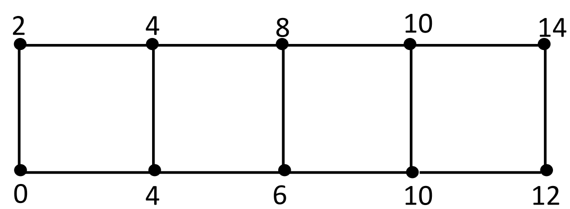

Figure 2.5

Theorem 2.13 The graph $S(B_{n,n})$ is an even vertex equitable even graph.

Proof: Let $V(B_{n,n}) = \{u, v\} \cup \{v_i, u_i : 1 \le i \le n\}$ and $E(B_{n,n}) = \{uv\} \cup \{uu_i, vv_i : 1 \le i \le n\}$. Let v_i' be the newly added vertex between v and v_i. Let u_i' be the newly added vertex between u and u_i. Let w be the newly added vertex between u and v. Then $S(B_{n,n})$ is of order $4n+3$ and size $4n+2$.

Define $f: V\left(S(B_{n,n})\right) \to A = \{0,2,4,\dots,4n+2\}$ as follows:

$$f(u) = 2 ;$$

$f(w) = f(v) = 4n$;

$f(u'_1) = 2$;

$f(u_i) = 4i - 4$; $1 \leq i \leq n$

$f(v_i) = 4i + 2$; $1 \leq i \leq n$

$f(u'_i) = 4i - 4$; $2 \leq i \leq n$

$f(v'_i) = 4i + 2$; $1 \leq i \leq n$

It can be easily verified that the induced edge labels of $S(B_{n,n})$ are $2,4,6, \dots ,8n + 4$.

Thus, $|v_f(a) - v_f(b)| \leq 1$ for all $a, b \in A$.

Hence the graph $S(B_{n,n})$ is an even vertex equitable even graph.

Theorem 2.14 The graph $S(L_n)$ is an even vertex equitable even graph.

Proof: Let $u_1, u_2, u_3, \dots, u_n$ and $v_1, v_2, v_3, \dots, v_n$ be the vertices of the ladder L_n. Let v'_i be the newly added vertex between v_i and v_{i+1}.Let u'_i be the newly added vertex between u_i and u_{i+1}. Let w_i be the newly added vertex between v_i and u_i. Then $S(L_n)$ is of order $5n - 2$ and size $6n - 4$.

Define $f: V(S(L_n)) \to A = \{0,2,4, \dots ,6n - 4\}$ as follows:

$$f(u_i) = \begin{cases} 6i - 4 & if \ i \ is \ odd \\ 6i - 6 & if \ i \ is \ even \end{cases} ; \ 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} 6i - 6 & if \ i \ is \ odd \\ 6i - 4 & if \ i \ is \ even \end{cases} ; \ 1 \leq i \leq n$$

$$f(u'_i) = \begin{cases} 6i - 2 & if \ i \ is \ odd \\ 6i + 2 & if \ i \ is \ even \end{cases} ; 1 \leq i \leq n - 1$$

$$f(v'_i) = \begin{cases} 6i + 2 & if \ i \ is \ odd \\ 6i - 2 & if \ i \ is \ even \end{cases} ; 1 \leq i \leq n - 1$$

$f(w_1) = 2$;

$f(w_i) = 6i - 6$; $2 \leq i \leq n$

It can be easily verified that the induced edge labels of $S(L_n)$ are $2,4,6,\ldots,12n-8$.

Thus, $|v_f(a)-v_f(b)| \leq 1$ for all $a, b \in A$.

Hence the graph $S(L_n)$ is an even vertex equitable even graph.

Example 2.15 An even vertex equitable even labeling of $S(L_5)$ is shown in Figure 2.6

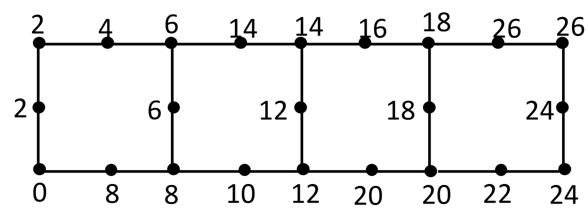

Figure 2.6

Theorem 2.16 The graph $L_n \odot K_1$ is an even vertex equitable even graph.

Proof: Let $u_1, u_2, u_3, \ldots, u_n$ and $v_1, v_2, v_3, \ldots, v_n$ be the vertices of the ladder L_n. Let u_i' and v_i' be the new vertices adjacent with u_i and v_i respectively. Then $L_n \odot K_1$ is of order $4n$ and size $5n-2$.

Define $f: V(L_n \odot K_1) \to A = \begin{cases} 0,2,4,\ldots,5n-1 & if\ \ 5n-2\ is\ odd \\ 0,2,4,\ldots,5n-2 & if\ \ 5n-2\ is\ even \end{cases}$ as follows:

$$f(u_{2i-1}) = 10i - 8\ ; 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$$

$$f(u_{2i}) = 10i - 4\ ; 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$

$$f(v_{2i-1}) = 10i - 8\ ; 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$$

$$f(v_{2i}) = 10i - 2\ ; 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$

$$f(u'_{2i-1}) = 10i - 10\ ; 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$$

$$f(u'_{2i}) = 10i - 4\ ; 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$

$$f(v'_{2i-1}) = 10i - 6\ ; 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$$

$$f(v'_{2i}) = 10i - 2\ ; 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$

It can be easily verified that the induced edge labels of $L_n \odot K_1$ are $2,4,6, \ldots ,10n - 4$.

Thus, $\left|v_f(a) - v_f(b)\right| \leq 1$ for all $a, b \in A$.

Hence the graph $L_n \odot K_1$ is an even vertex equitable even graph.

Example 2.17 An even vertex equitable even labeling of $L_5 \odot K_1$ is shown in Figure 2.7

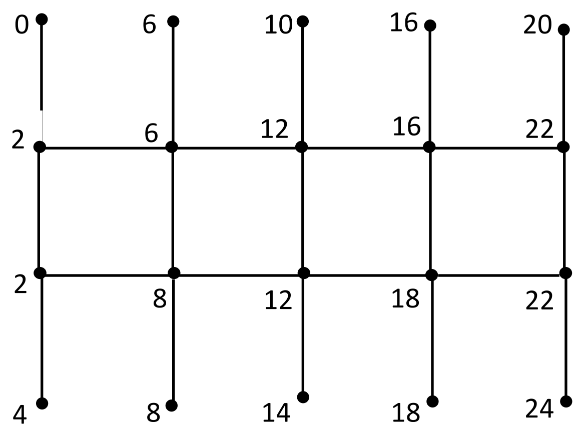

Figure 2.7

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